Fourth International Conference on Advanced COmputational Methods in ENgineering (ACOMEN 2008)
Editors: M. Hogge, R. Van Keer, L. Noels, L. Stainier, J.-P. Ponthot, J.-F. Remacle, E. Dick


# Algorithm for solving fluid-structure interaction problem numerically: application to cerebral aneurysm

S. Sy [1] and C.M. Murea [1]

[1] Laboratoire de Mathématiques, Informatique et Applications,
Université de Haute Alsace

4-6, rue des Frères Lumière, 68093 Mulhouse Cedex, France.
e-mails: *soyibou.sy@uha.fr*, *cornel.murea@uha.fr*

**Abstract**

This paper deals with a inconditional semi-implicit algorithm for solving fluid-structure interaction problem numerically holding in cerebral aneurysm. At each time step, an optimization problem is solved by partitioned procedure, in order to get the continuity of stress as well as the continuity of velocity at the interface. Numerical results are presented.

## 1 Introduction

The cerebral aneurysm appears when the intercranial arterial wall dilates in anomaly way under divers factors. It create therefore a pocket where the blood accumulate. The consequence of the cerebral aneurysm is often their rupture and intercranial hemorrhage with an associated high mortality rate (see [1]). There are some paper dealing with fluid-structure interaction between the blood and the wall aneurysm, see for example [2] and [3]. In [3], the authors, describe the flow dynamics and arterial wall interaction of a terminal aneurysm of simplified basilar artery and they compute its wall shear stress, pressure, effective stress and wall deformation. In [2], the fluid-structure interaction of a patient specific cerebral aneurysm located in the left middle cerebral bifurcation for high blood pressure is studied.

The geometry of cerebral aneurysm considered here is similar to one used in the article [4]. In the present work, we present a fast semi-implicit algorithm for solving numerically the interaction between the blood and the arterial wall in cerebral aneurysm. The term semi-implicit means that the velocity, the pressure of the fluid and the displacement of the structure are computed in implicit way, while the interface between the fluid and the structure is treated in explicit way. In [5], we have showed that this algorithm is inconditional stable, using the energy estimates. The implementation of the algorithm is the following: at each time step we have to solve a least square problem based on the Broyden, Fletcher, Goldford, Shano (BFGS) algorithm by partitioned procedure in order to get the continuity of the stress as well as the continuity of velocity at the interface. The major importance to work with this algorithm is that it use a fixed mesh during the optimization problem, that reduce considerably the time of computation.

## 2 Setting problem

The fluid and the structure models are setting in two dimensional spaces. We are interested by fluid-structure interaction problems holding in cerebral aneurysm.

Let us denote by $\Omega^S$ the undeformed structure domain bounded by: the rigid sections $\Gamma_1$ and $\Gamma_2$, the upper section $\Gamma_3$, the lower section $\Sigma_0$. We also denote by $\Omega_0^F$ the initial fluid domain bounded by: the rigid section $\Sigma_1$, the inflow section $\Sigma_2$, the outflow section $\Sigma_3$ and the top boundary $\Sigma_0$, (see Figure 1). The boundary $\Sigma_0$ is common of both domains and it represents the fluid-structure interface. Under the action of the fluid stress, the structure will be deformed. At the time instant $t$, the fluid occupies the domain $\Omega_t^F$ bounded by the moving interface $\Sigma_t$ and the rigid boundary $\Sigma_1 \cup \Sigma_2 \cup \Sigma_3$.

We assume that the fluid is viscous, Newtonian and incompressible and it is governed by the Navier-Stokes equations. We also assume that the structure is governed by the linear elasticity equations. To couple the fluid and the structure, we impose the continuity of velocity and the equality of stress at the interface. At each time $t \in [0,T]$, we are interested to know: the fluid velocity $\mathbf{v}(t) = (\mathbf{v}_1(t), \mathbf{v}_2(t))^T : \Omega_t^F \longrightarrow \mathbb{R}^2$, the fluid pressure $p(t) : \Omega_t^F \longrightarrow \mathbb{R}$ and the structure displacement $\mathbf{u}(t) = (\mathbf{u}_1(t), \mathbf{u}_2(t))^T : \Omega^S \longrightarrow \mathbb{R}^2$.

We use the ALE (Arbitrary Lagrangian Eulerian) coordinates to include the mesh velocity in the fluid equations, see for example [6]. Let $\widehat{\Omega}^F$ be the reference fixed domain and let $\mathcal{A}_t, t \in [0,T]$ be a family of transformations such that:

$$\mathcal{A}_t(\widehat{\mathbf{x}}) = \widehat{\mathbf{x}},\ \forall\, \widehat{\mathbf{x}} \in \partial\Omega_t^F \setminus \Sigma_t,\ \mathcal{A}_t(\widehat{\Omega}^F) = \Omega_t^F,$$

where $\widehat{\mathbf{x}} = (\widehat{\mathbf{x}}_1, \widehat{\mathbf{x}}_2)^T \in \widehat{\Omega}^F$ are the ALE coordinates and $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^T = \mathcal{A}_t(\widehat{\mathbf{x}})$ the Eulerian coordinates. We denote the domain velocity of the fluid by:

$$\boldsymbol{\vartheta}(\mathbf{x}, t) = \frac{\partial \mathcal{A}_t}{\partial t}(\widehat{\mathbf{x}}, t)$$

and the ALE time derivative of the fluid velocity by:

$$\left.\frac{\partial \mathbf{v}}{\partial t}\right|_{\widehat{\mathbf{x}}}(\mathbf{x}, t) = \frac{\partial \widehat{\mathbf{v}}}{\partial t}(\widehat{\mathbf{x}}, t).$$

We assume that the fluid-structure interaction is governed by the following equations:

*Navier-Stokes*

$$\begin{aligned}
\rho^F \left( \left.\frac{\partial \mathbf{v}}{\partial t}\right|_{\widehat{\mathbf{x}}} + ((\mathbf{v} - \boldsymbol{\vartheta}) \cdot \nabla)\, \mathbf{v} \right) - 2\mu^F \nabla \cdot \epsilon(\mathbf{v}) + \nabla p &= \mathbf{f}^F,\ \Omega_t^F \times (0,T] && (1)\\
\nabla \cdot \mathbf{v} &= 0,\ \Omega_t^F \times [0,T] && (2)\\
\sigma^F \mathbf{n}^F &= \mathbf{h}_{in},\ \Sigma_2 \times (0,T] && (3)\\
\sigma^F \mathbf{n}^F &= \mathbf{h}_{out},\ \Sigma_3 \times (0,T] && (4)\\
\mathbf{v} &= 0,\ \Sigma_1 \times (0,T] && (5)\\
\mathbf{v}(X,0) &= \mathbf{v}^0(X),\ \Omega_0^F && (6)
\end{aligned}$$

*linear elasticity*

$$\begin{aligned}
\rho^S \frac{\partial^2 u}{\partial t^2} - \nabla \cdot \sigma^S &= \mathbf{f}^S, \quad \text{in } \Omega^S \times (0,T] && (7)\\
\mathbf{u} &= 0, \quad \text{on } \Gamma_1 \cup \Gamma_2 \times (0,T] && (8)\\
\sigma^S \mathbf{n}^S &= 0, \quad \text{on } \Gamma_3 \times (0,T] && (9)\\
\mathbf{u}(X,0) &= \mathbf{u}^0(X), \quad \text{in } \Omega^S && (10)\\
\frac{\partial \mathbf{u}}{\partial t}(X,0) &= \dot{\mathbf{u}}^0(X), \quad \text{in } \Omega^S && (11)
\end{aligned}$$

*interface conditions*

$$\begin{aligned}
\mathbf{v}(X + \mathbf{u}(X,t), t) &= \frac{\partial \mathbf{u}}{\partial t}(X,t), \quad \text{on } \Sigma_0 \times (0,T] && (12)\\
(\sigma^F \mathbf{n}^F)_{(X+\mathbf{u}(X,t),t)}\,\omega &= -(\sigma^S \mathbf{n}^S)_{(X,t)}, \quad \text{on } \Sigma_0 \times (0,T]. && (13)
\end{aligned}$$

The following notations are using in above equations :

$$\epsilon(\mathbf{v}) = \frac{1}{2}\Big(\nabla \mathbf{v} + (\nabla \mathbf{v})^T\Big),\ \sigma^F = -p\mathbb{I}_2 + 2\mu^F \epsilon(\mathbf{v}),\ \sigma^S = \lambda^S(\nabla \cdot \mathbf{u})\mathbb{I}_2 + 2\mu^S \epsilon(\mathbf{u}),$$

$\rho^F > 0$ is the mass density of the fluid, $\rho^S > 0$ is the mass density of the structure), $\mu^F$ is the viscosity of the fluid, $\mu^S$, and $\lambda^S$ are the Lamé coefficients). $\mathbf{f}^F = (\mathbf{f}_1^F, \mathbf{f}_2^F)$ are the applied volume forces of the fluid and $\mathbf{f}^S = (\mathbf{f}_1^S, \mathbf{f}_2^S)$ the applied forces volume on the structure. $\mathbf{h}_{in}$ and $\mathbf{h}_{out}$ are the prescribed boundary stress on $\Sigma_2$ and on $\Sigma_3$. $\omega = \|\mathrm{cof}(\nabla \mathbb{T}_u)\mathbf{n}^S\|_{\mathbb{R}^2}$, where $\mathbb{T}_u$ is the application from $\Gamma_0$ in $\Gamma_t$ defined by: $\mathbb{T}_u(X) = X + \mathbf{u}(X,t)$, $\mathrm{cof}(\nabla \mathbb{T}_u)$ is co-factor matrix of $\nabla \mathbb{T}_u$ and $\mathbf{n}^S = (\mathbf{n}_1^S, \mathbf{n}_2^S)$ is the unit outward normal to $\Gamma_0$.

# 3 Time discretization and algorithm

## 3.1 Time discretization

Let $N \in \mathbb{N}^*$ be the number of time steps and we denote by $\Delta t = \dfrac{T}{N}$ the step time. We set $t_n = n\Delta t$ for $n = 0, \cdots, N$ the subdivision of $[0,T]$. We suppose that $\mathbf{f}^F : [0,T] \longrightarrow (L^2(\Omega_t^F))^2$, $\mathbf{h}_{in} : [0,T] \longrightarrow L^2(\Sigma_2)$, $\mathbf{h}_{out} : [0,T] \longrightarrow L^2(\Sigma_3)$ and $\mathbf{f}^S : [0,T] \longrightarrow (L^2(\Omega^S))^2$ are continuous maps. We set $\mathbf{f}^n = \mathbf{f}^F(n\Delta t)$, $\mathbf{h}_{in}^n = \mathbf{h}_{in}(n\Delta t)$, $\mathbf{h}_{out}^n = \mathbf{h}_{out}(n\Delta t)$ and $\mathbf{g}^n = \mathbf{f}^S(n\Delta t)$ and we define $\mathbf{u}^n$ the approximation of $\mathbf{u}(n\Delta t)$.

We consider an implicit Euler scheme for the time derivative and a linearization of the convection term for the the fluid equations and we employ a $\theta$-centered scheme of second-order in time for the structure equations. Let us set $\widehat{\Omega}^F = \Omega_n^F$ and we define $\boldsymbol{\vartheta}^n$ the velocity of the fluid domain to be solution of:

$$\left\{\begin{array}{lcll} \Delta_{\widehat{\mathbf{x}}}\boldsymbol{\vartheta}^n & = & 0, & \Omega_n^F \\ \boldsymbol{\vartheta}^n & = & 0, & \partial\Omega_n^F \setminus \Gamma_n \\ \boldsymbol{\vartheta}^n & = & \mathbf{v}^n, & \Gamma_n, \end{array}\right. \tag{14}$$

where $\mathbf{v}^n$ is the fluid velocity at time $n$ on $\Omega_n^F$.
Under the assumption that $\Omega_n^F$ is Lipschitz, we have $\boldsymbol{\vartheta}^n \in (H^1(\Omega_n^F))^2$.

For all $n = 0, \cdots, N-1$, we denote by $\mathbb{T}_n$ the following map:

$$\begin{array}{rcl} \mathbb{T}_n(\widehat{\mathbf{x}_1}, \widehat{\mathbf{x}_2}) : \overline{\Omega}_n^F & \longrightarrow & \mathbb{R}^2 \\ (\widehat{\mathbf{x}}_1, \widehat{\mathbf{x}}_2) & \mapsto & (\widehat{\mathbf{x}}_1 + \Delta t\boldsymbol{\vartheta}_1^n, \widehat{\mathbf{x}}_2 + \Delta t\boldsymbol{\vartheta}_2^n). \end{array}$$

We set $\Omega_{n+1}^F = \mathbb{T}_n(\Omega_n^F)$ and $\Gamma_{n+1} = \mathbb{T}_n(\Gamma_n)$. The Jacobian of $\mathbb{T}_n$ is obtained by:

$$1 + \Delta t(\nabla_{\widehat{\mathbf{x}}} \cdot \boldsymbol{\vartheta}^n) + (\Delta t)^2\Big(\frac{\partial \boldsymbol{\vartheta}_1^n}{\partial \widehat{\mathbf{x}}_1} \cdot \frac{\partial \boldsymbol{\vartheta}_2^n}{\partial \widehat{\mathbf{x}}_2} - \frac{\partial \boldsymbol{\vartheta}_2^n}{\partial \widehat{\mathbf{x}}_1} \cdot \frac{\partial \boldsymbol{\vartheta}_1^n}{\partial \widehat{\mathbf{x}}_2}\Big).$$

Finally we define the map $\mathbb{T}$ as:

$$\mathbb{T} = \mathbb{T}_{n-1} \circ \mathbb{T}_{n-2} \circ \mathbb{T}_{n-3} \cdots \circ \mathbb{T}_0$$

and we can observe that $\Gamma_n = \mathbb{T}(\Gamma_0)$.

## 3.2 Algorithm

In order to integrate $(1)$-$(13)$ with respect to space, we define the following spaces of test functions:

$$\begin{array}{rcl} \widehat{W}_n^F & = & \{\widehat{\mathbf{w}}^F \in (H^1(\Omega_n^F))^2;\ \widehat{\mathbf{w}}^F = 0 \text{ on } \Sigma 1\} \\ \widehat{Q}_n^F & = & L^2(\Omega_n^F) \\ W^S & = & \{\mathbf{w}^S \in (H^1(\Omega^S))^2;\ \mathbf{w}^S = 0 \text{ on } \Gamma_1 \cup \Gamma_2\}. \end{array}$$

We assume that we know $\Omega_n^F$, $\mathbf{v}^n \in (L^2(\Omega_n^F))^2$, $\mathbf{u}^{n-1}, \mathbf{u}^n \in \left(L^2(\Omega^S)\right)^2$.
**Step** 1: Find $\boldsymbol{\vartheta}^n \in (H^1(\Omega_n^F))^2$ solution of (14)
**Step** 2: Find $\widehat{\mathbf{v}}^{n+1} \in \widehat{W}_n^F$, $\widehat{p}^{n+1} \in \widehat{Q}_n^F$, $\mathbf{u}^{n+1} \in W^S$ with

$$\widehat{\mathbf{v}}^{n+1} \circ \mathbb{T} = \frac{\mathbf{u}^{n+1} - \mathbf{u}^{n-1}}{2\Delta t}, \quad \text{on } \Sigma_0$$

such that:

$$\begin{aligned}
&\rho^F \int_{\Omega_n^F} \frac{(\widehat{\mathbf{v}}^{n+1} - \mathbf{v}^n)}{\Delta t} \cdot \widehat{\mathbf{w}}^F + \rho^F \int_{\Omega_n^F} (((\mathbf{v}^n - \boldsymbol{\vartheta}^n) \cdot \nabla_{\widehat{\mathbf{x}}}) \widehat{\mathbf{v}}^{n+1}) \cdot \widehat{\mathbf{w}}^F \\
&+ \frac{1}{2} \int_{\Omega_n^F} \delta(\widehat{\mathbf{x}}) \widehat{\mathbf{v}}^{n+1} \cdot \widehat{\mathbf{w}}^F + 2\mu^F \int_{\Omega_n^F} \epsilon_{\widehat{\mathbf{x}}}(\widehat{\mathbf{v}}^{n+1}) : \epsilon_{\widehat{\mathbf{x}}}(\widehat{\mathbf{w}}^F) \\
&- \int_{\Omega_n^F} \widehat{p}^{n+1} (\nabla_{\widehat{\mathbf{x}}} \cdot \widehat{\mathbf{w}}^F) - \int_{\Omega_n^F} \widehat{q} (\nabla_{\widehat{\mathbf{x}}} \cdot \widehat{\mathbf{v}}^{n+1}) \\
&+ \rho^S \int_{\Omega^S} \Big( \frac{\mathbf{u}^{n+1} - 2\mathbf{u}^n + \mathbf{u}^{n-1}}{\Delta t^2} \Big) \cdot \mathbf{w}^S \\
&+ a_S(\theta \mathbf{u}^{n+1} + (1 - 2\theta)\mathbf{u}^n + \theta \mathbf{u}^{n-1}, \mathbf{w}^S) \\
&= \int_{\Omega_n^F} \widehat{\mathbf{f}}^{n+1} \cdot \widehat{\mathbf{w}}^F + \int_{\Omega^S} \overline{\mathbf{g}}^{n+1} \cdot \mathbf{w}^S + \int_{\Sigma_1} \mathbf{h}_{in}^{n+1} \cdot \mathbf{w}^F + \int_{\Sigma_3} \mathbf{h}_{out}^{n+1} \cdot \mathbf{w}^F,
\end{aligned} \tag{15}$$

for any $\widehat{\mathbf{w}}^F \in \widehat{W}_n,\ \widehat{q} \in \widehat{Q}_n^F,\ \mathbf{w}^S \in W^S$ with $\mathbf{w}^S = \mathbf{w}^F \circ \mathbb{T}$ on $\Sigma_0$,
where

$$\widehat{\mathbf{f}}^{n+1} = \mathbf{f}^{n+1} \circ \mathbb{T} \text{ et } \overline{\mathbf{g}}^{n+1} = \theta \mathbf{g}^{n+1} + (1 - 2\theta)\mathbf{g}^n + \theta \mathbf{g}^{n-1}$$

and

$$\delta(\widehat{\mathbf{x}}) = \rho^F \Delta t \Big( \frac{\partial \boldsymbol{\vartheta}_1^n}{\partial \widehat{\mathbf{x}}_1} \cdot \frac{\partial \boldsymbol{\vartheta}_2^n}{\partial \widehat{\mathbf{x}}_2} - \frac{\partial \boldsymbol{\vartheta}_2^n}{\partial \widehat{\mathbf{x}}_1} \cdot \frac{\partial \boldsymbol{\vartheta}_1^n}{\partial \widehat{\mathbf{x}}_2} \Big).$$

Using the finite element method, we can directly solve the monolithic linear system (15), in this case the continuity of the velocity at the interface must be satisfied as an essential boundary condition. So the fluid test functions must coincide to the structure test functions at the interface, that implies some constraints for triangulation of the fluid and structure domains as well as in the choice of the finite elements. Other methods have been developed to solve this kind of problem, for example, in [7] the problem is solved by Augmented Lagrangian method where the continuity of the velocity was treated by a Lagrange multiplier and the numerical results presented show that the continuity of the velocity is not very well respected since the error between the velocity is $0.45$.
The method that we use here to solve the coupled problem is based on partitioned procedure, which is very often used to solve the fluid-structure interaction problem. As in [8], where an implicit algorithm is presented, we use the same least square method based on BFGS method in order to identify the stress at the interface.

The structure problem (the weak formulation corresponding to (7)-(11)) is solved numerically by modal decomposition and a $\theta$-scheme. We set $\mathbf{u}(t) = \sum_{i \geq 1} q_i(t)\phi_i$, where $\phi_i$ is the eigenfunction associated to the eigenvalue $\lambda_i$. Find $q_i^{n+1}$ such that

$$\frac{q_i^{n+1} - 2q_i^n + q_i^{n-1}}{(\Delta t)^2} + \lambda_i(\theta q_i^{n+1} + (1 - 2\theta)q_i^n + \theta q_i^{n-1}) = \theta \alpha_i^{n+1} + (1 - 2\theta)\alpha_i^n + \theta \alpha_i^{n-1},$$

where $\alpha_i^{n+1} = \alpha_i(t_{n+1}) = \int_{\Omega^S} \mathbf{f}^S(t_{n+1})\phi_i + \int_{\Gamma_0} (\sigma^S \mathbf{n}^S)\phi_i$.

**Algorithm for solving the fluid-structure coupled problem at time instant** $t_{n+1}$

**Step 1** Compute the mesh velocity $\boldsymbol{\vartheta}^n$ from $(14)$.

**Step 2** Assembling the finite element matrix of fluid problem (weak form corresponding to $(1)$-$(6)$) using the mesh $\mathcal{T}^n$ obtained at the previous time step. Get a LU factorization of the matrix.

**Step 3** Solve the fluid-structure coupled problem using the fluid frozen mesh $\mathcal{T}^n$ by BFGS algorithm (see [8]).

$$\alpha^{n+1} \in \arg\min_{\alpha \in \mathbb{R}^m} J(\alpha),$$

where $J(\alpha) = \dfrac{1}{2}\|\alpha - \beta\|^2_{\mathbb{R}^m}$ with $\beta = \displaystyle\int_{\Omega^S} \mathbf{f}^S(t)\cdot\phi_i - \int_{\Gamma_0}(\sigma^F \mathbf{n}^F)\cdot\phi_i\omega(X,t).$

**Step 4** Build mesh $\mathcal{T}^{n+1}$, as the image of $\mathcal{T}^n$ by the map $\widehat{x} \mapsto \widehat{x} + \Delta t\boldsymbol{\vartheta}^n(\widehat{x})$ and save the mesh $\mathcal{T}^{n+1}$, the velocity $\mathbf{v}^{n+1}(\mathbf{x}) = \widehat{\mathbf{v}}^{n+1}(\widehat{\mathbf{x}})$, etc.

**Remark 1** *Contrary to the implicit strategy, the semi-implicit one use a fixed fluid mesh during the iterative method for solving the optimization problem at the* ***Step 3****, which reduce considerable the computational time.*

# 4 Numerical results

*Physical parameters*

We consider the following data for the computation: the length of the inflow and the outflow sections is 3 mm, the length of the rigid section $\Sigma_2$ is 5 mm and for the interface $\Sigma_4$, we take an arc with diameter equal to 6 mm. The viscosity of the fluid was fixed to be $\mu = 0.003\ \frac{g}{cm\cdot s}$, its density $\rho^F = 1\ \frac{g}{cm^3}$ and the volume force in fluid is $\mathbf{f}^F = (0,\ 0)^T$. The prescribed boundary stress at the outflow is $\mathbf{h}_{out}(x,t) = (0,\ 0)$ and at the inflow is

$$\mathbf{h}_{in}(x,t) = \begin{cases} (10^3(1-\cos(2\pi t/0.025)),\ 0), & x \in \Sigma_2, 0 \le t \le 0.025 \\ (0,\ 0), & x \in \Sigma_2, 0.025 \le t \le T. \end{cases}$$

The elastic wall is the arc $\Sigma_4$ with diameter 6 mm. The Young modulus $E = 3\cdot 10^6\ \frac{g}{cm\cdot s^2}$, the Poisson ratio $\nu = 0.3$, the mass density $\rho^S = 1.1\frac{g}{cm^3}$ and the volume force is $\mathbf{f}^S = (0,\ 0)^T$. The Lamé's coefficients are computed by the formulas:

$$\lambda^S = \frac{\nu^S E}{(1-2\nu^S)}(1+\nu^S), \qquad \mu^S = \frac{E}{2(1+\nu^S)}.$$

*Numerical parameters*

The numerical tests have been performed using FreeFem++ (see [9]). We have used for the structure a reference mesh of $60$ triangles and $62$ vertices and for the fluid a reference mesh of $1615$ triangles and $881$ vertices. The compatibility of meshes are not necessary verified at the interface (see Figure 2). For the approximation of the fluid velocity and pressure, we have used the triangular finite element $\mathbb{P}_1 + bubble$ and $\mathbb{P}_1$ respectively. The finite element $\mathbb{P}_1$ was employed in order to solve the eigenproblem of the structure. Only the first $m = 3$ modes have been considered. The first eigenvalues are $\lambda_{1,h} = 1845190$, $\lambda_{2,h} = 7440200$ and $\lambda_{3,h} = 24656000$. The real parameter in the $\theta$-centered scheme was chosen to be $\theta = 0.3$.

*Stopping criteria*

At each time step, the optimization problem have been solved by the BFGS algorithm. The final values of the cost function are less than $6.10^{-10}$. In other words, the continuity of the velocity at the interface holds at every time step, while the error between the fluid and structure stress at the interface is less than $6.10^{-10}$. In [7], the fluid-structure coupled problem is solved by the Augmented Lagrangian Method and at every time step the continuity of the stress at the interface holds, while the error in the $L^2$

norm between the fluid and structure velocity at the interface is less than $0.45$. Consequently, the boundary conditions at the interface are verified more precisely when we solve by the BFGS algorithm. We have used the FreeFem++ to implement the BFGS algorithm which use the stopping criteria: $\|\nabla J\| \leq \epsilon$ or the number of iterations reaches a maximal value $nbiter$. We have performed the computation with $\epsilon = 10^{-4}$ and $nbiter = 10$. We set to $5$ maximal number of the iterations for the time search. We compute $\nabla J(\alpha)$ by finite difference scheme:

$$\frac{\partial J}{\partial \alpha_k}(\alpha) = \frac{J(\alpha + \Delta\alpha_k e_k) - J(\alpha)}{\Delta\alpha_k}$$

where $e_k$ is the $k$-th vector of the canonical base of $\mathbb{R}^m$. Thus, $m + 1 = 4$ calls of the cost function are needed to compute the gradient.

**Remark 2** *The fluid mesh and the structure mesh are necessary compatible at the interface, see Figure (2). The fluid velocity (see Figure (4) as well as the fluid pressure (see Figure (3) are plotted, in order to describe the fluid behaviour in the cerebral aneurysm.*

# 5 Conclusion

In this paper, we have applied an inconditional semi-implicit algorithm, that we developed, to solve the interaction between the blood and the arterial wall in cerebral aneurysm. At each time step, an optimization problem is solved by partitioned procedure based in BFGS algorithm in order to get the continuity of velocity as well as the continuity of the stress at the interface.

# References


[1] Linn, F.H., Rinkel, G.J., Algra, A. and Van Gijn, J., Influence of subarachnoid haemorrhage: role of region, year and rate of computed tomography: a metaanalysis, *Stokes*, 27(4):625-629, 1996.

[2] Torii, R., Oshima, M., Kobayashi, T., Takagi, K. and Tezduyar, T.E., Fluid-structure interaction modeling of aneurysmal condition with high and normal blood pressure, *Mech.*, 38:482-490, 2006.

[3] Valencia, A. and Solis, F., Blood flow dynamics and arterial wall interaction in a saccular aneurysm model of the basilar artery, *Comput. Struct.*, 84:1326-1337, 2006.

[4] Tayfun E. Tezduyar, S. Sathe, T. Cragin, B. Nanna, Brian S. Conklin, J. Pausewang and M. Schwaab, Modelling of fluid-structure interactions with the space-time finite elements: Arterial fluid mechanics. *Int. J. Numer. Meth. Fluids*, 54:901-922, 2007.

[5] S. Sy, C.M. Murea, A stable time advcing scheme for solving fluid-structure interaction problem, submitted

[6] A. Quarteroni, L. Formaggia, Mathematical modelling and numerical simulation of the cardiovascular system, in P.G. Ciarlet (Ed.), Handbook of numerical analysis, Vol. XII, North-Holland, Amsterdam, 3–127, 2004.

[7] C.M. Murea, A semi-implicit algorithm based on the Augmented Lagragian Method for fluid-structure interface. Proceeding of ENUMAT 07 Graz, Steptember 10-17, 2007, submitted.

[8] C.M. Murea, Numerical simulation of a pulsatile flow through a flexible channel. *ESAIM: Math. Model. Numer. Anal.*, 40:1101–1125, 2006.

[9] F. Hecht, O. Pironneau, A. Le Hyaric, K. Ohtsuka, FreeFem++: http://www.freefem++.org/ff++.

# 6 Figures

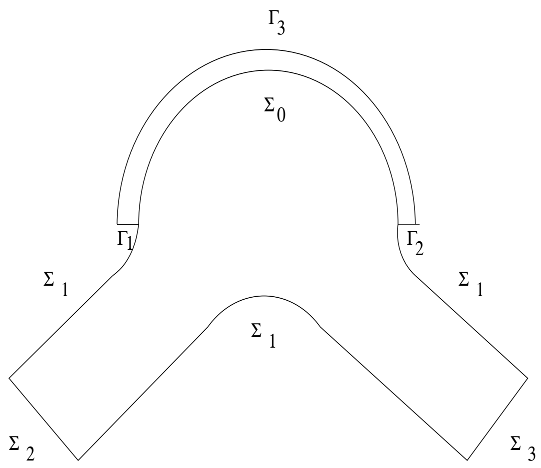


Figure 1: The fluid-structure domain.

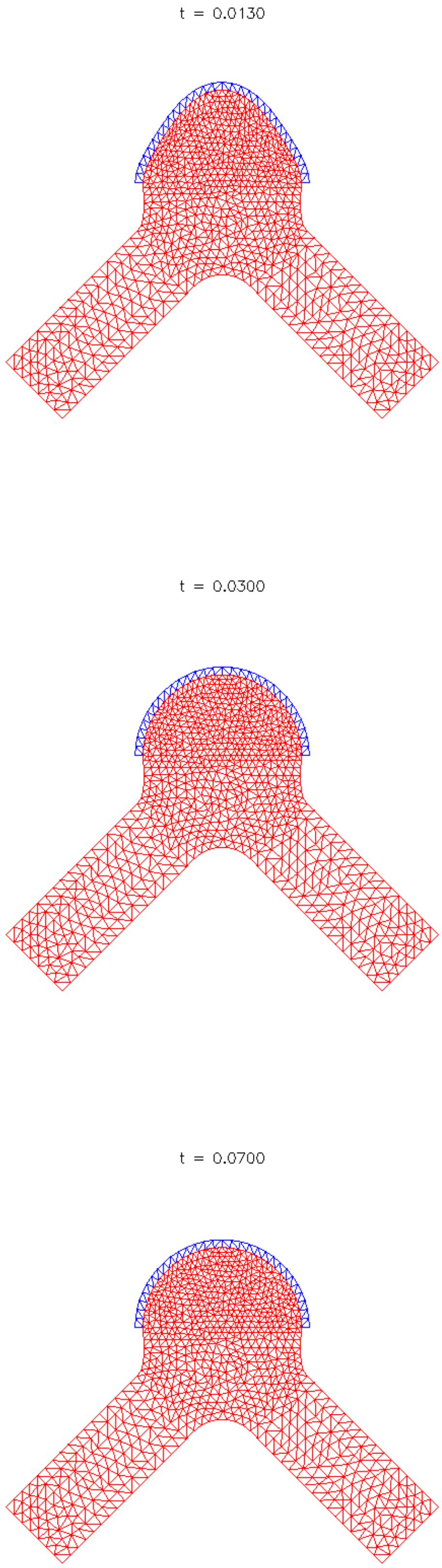


Figure 2: Fluid and structure meshes at time instant $t = 0.015$ (top), $t = 0.030$ (middle), $t = 0.070$ (bottom)

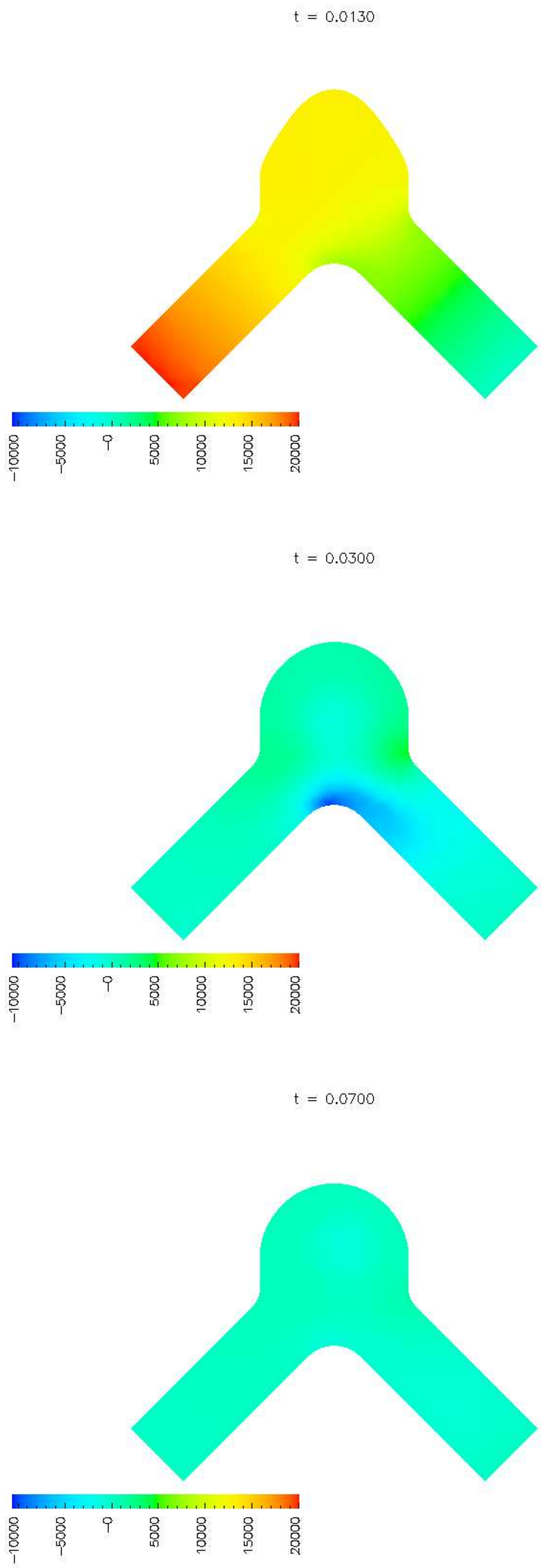


Figure 3: Fluid pressure $[\frac{dynes}{cm^2}]$ at time instant $t = 0.015$ (top), $t = 0.030$ (middle), $t = 0.070$ (bottom)

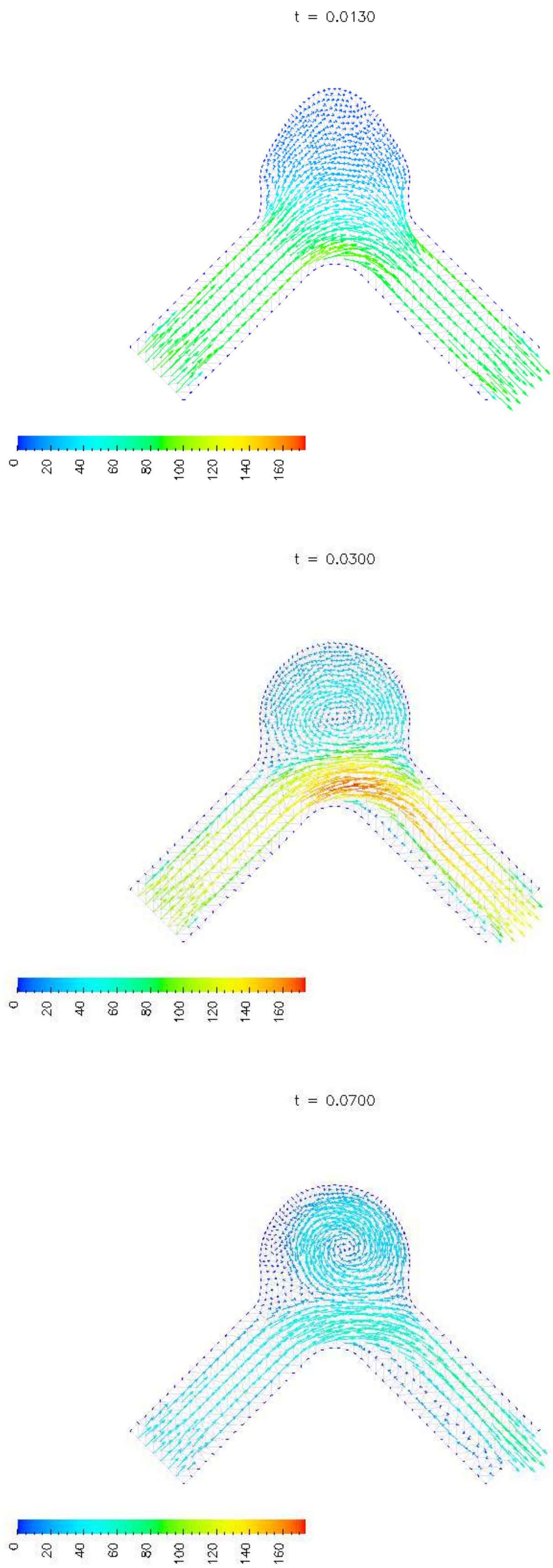


Figure 4: Fluid velocity $[cm/s]$ at time instant $t = 0.015$ (top), $t = 0.030$ (middle), $t = 0.070$ (bottom)