A semi-implicit algorithm based on the Augmented Lagrangian Method for fluid-structure interaction

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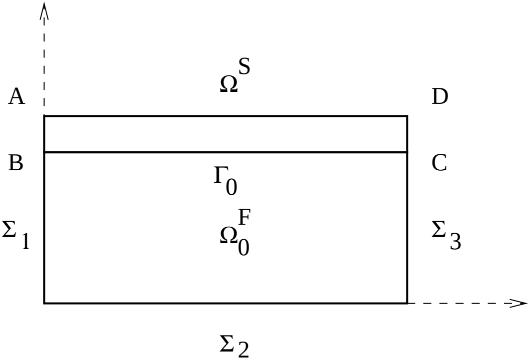
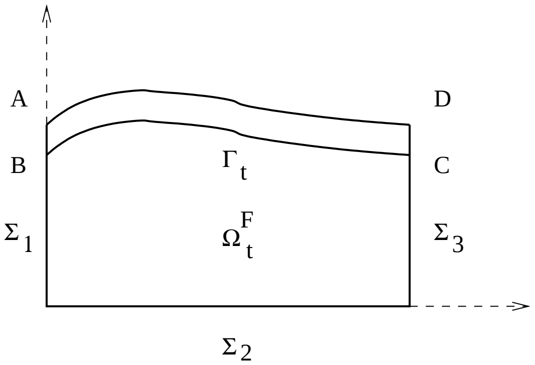
Initial geometrical configuration



$$\partial\Omega_0^F = \Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \cup \Gamma_0.$$

Intermediate geometrical configuration



$$\partial\Omega_t^F = \Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \cup \Gamma_t .$$

Linear elasticity equations

Find the displacement $\mathbf{u} = (u_1, u_2)^T : \Omega^S \times [0, T] \to \mathbb{R}^2$ of the structure such that

$$
\begin{aligned}
\rho^S \frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla \cdot \sigma^S &= \mathbf{f}^S, \quad \text{in } \Omega^S \times (0, T) \\
\sigma^S &= \lambda^S (\nabla \cdot \mathbf{u}) \mathbb{I}_2 + 2\mu^S \epsilon(\mathbf{u}) \\
\epsilon(\mathbf{u}) &= \frac{1}{2}\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right) \\
\mathbf{u} &= 0, \quad \text{on } \Gamma_D \times (0, T) \\
\sigma^S \mathbf{n}^S &= 0, \quad \text{on } \Gamma_N \times (0, T)
\end{aligned}
$$

$$\Gamma_D = [AB] \cup [CD], \quad \Gamma_N = [DA].$$

Navier-Stokes equations

Find the velocity $\mathbf{v}$ and the pressure p of the fluid such that

$$
\begin{aligned}
\rho^F \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) - \nabla \cdot \sigma^F &= \mathbf{f}^F, \quad \forall t \in (0, T), \forall \mathbf{x} \in \Omega_t^F \\
\nabla \cdot \mathbf{v} &= 0, \quad \forall t \in (0, T), \forall \mathbf{x} \in \Omega_t^F \\
\sigma^F &= -p \mathbb{I}_2 + 2\mu^F \epsilon(\mathbf{v}) \\
\sigma^F \mathbf{n}^F &= \mathbf{h}_{in}, \quad \text{on } \Sigma_1 \times (0, T) \\
\sigma^F \mathbf{n}^F &= \mathbf{h}_{out}, \quad \text{on } \Sigma_3 \times (0, T) \\
\mathbf{v} \cdot \mathbf{n}^F &= 0, \quad \text{on } \Sigma_2 \times (0, T) \\
\boldsymbol{\tau}^F \cdot \left(\sigma^F \mathbf{n}^F \right) &= 0, \quad \text{on } \Sigma_2 \times (0, T)
\end{aligned}
$$

Interface and initial conditions

The interface Γ_t is the image of the boundary Γ_0 by the map

$$\mathbb{T}(\mathbf{X}) = \mathbf{X} + \mathbf{u}(\mathbf{X}, t).$$

Interface conditions

$$\begin{aligned} \mathbf{v}(\mathbf{X} + \mathbf{u}(\mathbf{X}, t), t) &= \frac{\partial \mathbf{u}}{\partial t}(\mathbf{X}, t), \ \forall (\mathbf{X}, t) \in \Gamma_0 \times (0, T) \\ \left(\sigma^F \mathbf{n}^F\right)_{(\mathbf{X}+\mathbf{u}(\mathbf{X},t),t)} \omega(\mathbf{X}, t) &= -\left(\sigma^S \mathbf{n}^S\right)_{(\mathbf{X},t)}, \ \forall (\mathbf{X}, t) \in \Gamma_0 \times (0, T) \end{aligned}$$

where $\omega(\mathbf{X}, t) = \left\|\mathrm{cof}(\nabla \mathbb{T})\, \mathbf{n}^S\right\|_{\mathbb{R}^2}$.

Initial conditions

$$\begin{aligned} \mathbf{u}(\mathbf{X}, t = 0) &= \mathbf{u}^0(\mathbf{X}), \ \text{in } \Omega^S \\ \frac{\partial \mathbf{u}}{\partial t}(\mathbf{X}, t = 0) &= \dot{\mathbf{u}}^0(\mathbf{X}), \ \text{in } \Omega^S \\ \mathbf{v}(\mathbf{x}, t = 0) &= \mathbf{v}^0(\mathbf{x}), \ \text{in } \Omega_0^F \end{aligned}$$

Weak formulation of the structure

Find the displacement $\mathbf{u}$ of the structure

$$\int_{\Omega^S} \rho^S \frac{\partial^2 \mathbf{u}}{\partial t^2} \cdot \mathbf{w}^S + a_S\left(\mathbf{u}, \mathbf{w}^S\right) = \int_{\Omega^S} \mathbf{f}^S \cdot \mathbf{w}^S + \int_{\Gamma_0} \left(\sigma^S \mathbf{n}^S\right) \cdot \mathbf{w}^S$$

$\forall \mathbf{w}^S = 0$ on Γ_D

$$a_S\left(\mathbf{u}, \mathbf{w}^S\right) = \int_{\Omega^S} \lambda^S \left(\nabla \cdot \mathbf{u}\right)\left(\nabla \cdot \mathbf{w}^S\right) + \int_{\Omega^S} 2\mu^S \epsilon\left(\mathbf{u}\right) : \epsilon\left(\mathbf{w}^S\right)$$

Arbitrary Lagrangian Eulerian (ALE) framework. Notations

Let $\widehat{\Omega}^F$ be a reference fixed domain.
Let $\mathcal{A}_t$, $t \in [0, T]$ be a family of transformations such that

$$\begin{aligned} \mathcal{A}_t(\widehat{\mathbf{x}}) &= \widehat{\mathbf{x}}, \quad \forall \widehat{\mathbf{x}} \in \Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \\ \mathcal{A}_t(\Gamma_0) &= \Gamma_t \\ \mathcal{A}_t\left(\widehat{\Omega}^F\right) &= \Omega_t^F \end{aligned}$$

$\widehat{\mathbf{x}} = (\widehat{x}_1, \widehat{x}_2)^T \in \widehat{\Omega}^F$ the ALE and $\mathbf{x} = (x_1, x_2)^T = \mathcal{A}_t(\widehat{\mathbf{x}})$ the Eulerian coordinates.
Let v be the velocity of the fluid in the Eulerian coordinates. The corresponding function in the ALE framework $\widehat{v} : \widehat{\Omega}^F \times [0, T] \to \mathbb{R}^2$ is defined by $\widehat{v}(\widehat{\mathbf{x}}, t) = v(\mathcal{A}_t(\widehat{\mathbf{x}}), t) = v(\mathbf{x}, t)$.
We denote the ALE time derivative by $\left.\frac{\partial v}{\partial t}\right|_{\widehat{\mathbf{x}}}(\mathbf{x}, t) = \frac{\partial \widehat{v}}{\partial t}(\widehat{\mathbf{x}}, t)$ and the domain velocity by $\boldsymbol{\vartheta}(\mathbf{x}, t) = \frac{\partial \mathcal{A}_t}{\partial t}(\widehat{\mathbf{x}})$.

Navier-Stokes equations in the ALE framework

Find the velocity $\mathbf{v}$ and the pressure p of the fluid such that

$$
\begin{aligned}
\rho^F \left(\left.\frac{\partial \mathbf{v}}{\partial t}\right|_{\widehat{\mathbf{x}}} + ((\mathbf{v} - \boldsymbol{\vartheta}) \cdot \nabla)\, \mathbf{v} \right) - 2\mu^F \nabla \cdot \epsilon(\mathbf{v}) + \nabla p &= \mathbf{f}^F, \\
\nabla \cdot \mathbf{v} &= 0,
\end{aligned}
$$

$\forall t \in (0, T), \forall \mathbf{x} \in \Omega_t^F$

$$
\begin{aligned}
\sigma^F \mathbf{n}^F &= \mathbf{h}_{in}, \quad \text{on } \Sigma_1 \times (0, T) \\
\sigma^F \mathbf{n}^F &= \mathbf{h}_{out}, \quad \text{on } \Sigma_3 \times (0, T) \\
\mathbf{v} \cdot \mathbf{n}^F &= 0, \quad \text{on } \Sigma_2 \times (0, T) \\
\boldsymbol{\tau}^F \cdot \left(\sigma^F \mathbf{n}^F \right) &= 0, \quad \text{on } \Sigma_2 \times (0, T)
\end{aligned}
$$

Weak formulation of the Navier-Stokes equations

Find $\mathbf{v} \cdot \mathbf{n}^F = 0$ on $\Sigma_2 \times (0, T)$ and p such that

$$\int_{\Omega_t^F} \rho^F \left.\frac{\partial \mathbf{v}}{\partial t}\right|_{\widehat{\mathbf{x}}} \cdot \mathbf{w}^F + \int_{\Omega_t^F} \rho^F \left(\left(\left(\mathbf{v} - \boldsymbol{\vartheta}\right) \cdot \nabla\right) \mathbf{v}\right) \cdot \mathbf{w}^F$$
$$+ a_F\left(\mathbf{v}, \mathbf{w}^F\right) + b_F\left(\mathbf{w}^F, p\right) = \int_{\Omega_t^F} \mathbf{f}^F \cdot \mathbf{w}^F$$
$$+ \int_{\Sigma_1} \mathbf{h}_{in} \cdot \mathbf{w}^F + \int_{\Sigma_3} \mathbf{h}_{out} \cdot \mathbf{w}^F + \int_{\Gamma_t} \left(\sigma^F \mathbf{n}^F\right) \cdot \mathbf{w}^F$$

$\forall \mathbf{w} \cdot \mathbf{n}^F = 0$ on $\Sigma_2 \times (0, T)$ and

$$b_F\left(\mathbf{v}, q\right) = 0, \forall q$$

$$a_F\left(\mathbf{v}, \mathbf{w}^F\right) = \int_{\Omega_t^F} 2\mu^F \epsilon\left(\mathbf{v}\right) : \epsilon\left(\mathbf{w}^F\right)$$
$$b_F\left(\mathbf{w}^F, q\right) = -\int_{\Omega_t^F} \left(\nabla \cdot \mathbf{w}^F\right) q$$

Weak formulation of the fluid-structure interaction

Structure

$$\ldots + \int_{\Gamma_0} \left(\sigma^S \mathbf{n}^S\right) \cdot \mathbf{w}^S, \quad \forall \mathbf{w}^S = 0 \text{ on } \Gamma_D \times (0, T)$$

Fluid

$$\ldots + \int_{\Gamma_t} \left(\sigma^F \mathbf{n}^F\right) \cdot \mathbf{w}^F, \quad \forall \mathbf{w}^F \cdot \mathbf{n}^F = 0 \text{ on } \Sigma_2 \times (0, T)$$

$$b_F\left(\mathbf{v}, q\right) = 0, \quad \forall q$$

Interface conditions

$$\begin{aligned}
\mathbf{v}\left(\mathbf{X} + \mathbf{u}\left(\mathbf{X}, t\right), t\right) &= \frac{\partial \mathbf{u}}{\partial t}\left(\mathbf{X}, t\right), \text{ on } \Gamma_0 \times (0, T) \\
\left(\sigma^F \mathbf{n}^F\right)_{(\mathbf{X}+\mathbf{u}(\mathbf{X},t),t)} \omega\left(\mathbf{X}, t\right) &= -\left(\sigma^S \mathbf{n}^S\right)_{(\mathbf{X},t)}, \text{ on } \Gamma_0 \times (0, T) \\
\mathbf{w}^F\left(\mathbf{X} + \mathbf{u}\left(\mathbf{X}, t\right), t\right) &= \mathbf{w}^S\left(\mathbf{X}, t\right), \text{ on } \Gamma_0 \times (0, T)
\end{aligned}$$

Partitioned procedures by Lagrange multiplier

Find $\mathbf{u} = 0$ on Γ_D, $\mathbf{v} \cdot \mathbf{n}^F = 0$ on Σ_2, p and $\boldsymbol{\eta}$ such that

$$\int_{\Omega^S} \rho^S \frac{\partial^2 \mathbf{u}}{\partial t^2} \cdot \mathbf{w}^S + a_S\left(\mathbf{u}, \mathbf{w}^S\right) - \int_{\Gamma_0} \widehat{\boldsymbol{\eta}} \cdot \mathbf{w}^S \omega = \int_{\Omega^S} \mathbf{f}^S \cdot \mathbf{w}^S$$

$\forall \mathbf{w}^S = 0$ on Γ_D

$$\int_{\Omega_t^F} \rho^F \left.\frac{\partial \mathbf{v}}{\partial t}\right|_{\widehat{x}} \cdot \mathbf{w}^F + \int_{\Omega_t^F} \rho^F \left(\left(\left(\mathbf{v} - \boldsymbol{\vartheta}\right) \cdot \nabla\right) \mathbf{v}\right) \cdot \mathbf{w}^F$$
$$+ a_F\left(\mathbf{v}, \mathbf{w}^F\right) + b_F\left(\mathbf{w}^F, p\right) + \int_{\Gamma_t} \boldsymbol{\eta} \cdot \mathbf{w}^F$$
$$= \int_{\Omega_t^F} \mathbf{f}^F \cdot \mathbf{w}^F + \int_{\Sigma_1} \mathbf{h}_{in} \cdot \mathbf{w}^F + \int_{\Sigma_3} \mathbf{h}_{out} \cdot \mathbf{w}^F$$

$\forall \mathbf{w}^F \cdot \mathbf{n}^F = 0$ on Σ_2

$$b_F\left(\mathbf{v}, q\right) = 0, \quad \forall q$$

$$\int_{\Gamma_0} \zeta \cdot \left(\widehat{\mathbf{v}} - \frac{\partial \mathbf{u}}{\partial t}\right) \omega = 0, \quad \forall \zeta$$

Newmark algorithm

$\Delta t > 0$ the time step, $t_{n+1} = (n+1)\Delta t$,
$\mathbf{u}^{n+1}$, $\dot{\mathbf{u}}^{n+1}$, $\ddot{\mathbf{u}}^{n+1}$ are approximations of
$\mathbf{u}(\cdot, t_{n+1})$, $\frac{\partial \mathbf{u}}{\partial t}(\cdot, t_{n+1})$ $\frac{\partial^2 \mathbf{u}}{\partial t^2}(\cdot, t_{n+1})$ respectively.

Find $\mathbf{u}^{n+1}$, $\dot{\mathbf{u}}^{n+1}$, $\ddot{\mathbf{u}}^{n+1}$ such that

$$\int_{\Omega^S} \rho^S \ddot{\mathbf{u}}^{n+1} \cdot \mathbf{w}^S + a_S\left(\mathbf{u}^{n+1}, \mathbf{w}^S\right) - \int_{\Gamma_0} \widehat{\boldsymbol{\eta}}^{n+1} \cdot \mathbf{w}^S \omega = \int_{\Omega^S} \mathbf{f}^S \cdot \mathbf{w}^S$$

$\forall \mathbf{w}^S = 0$ on Γ_D

$$\begin{aligned} \dot{\mathbf{u}}^{n+1} &= \dot{\mathbf{u}}^n + \Delta t\left((1-\delta)\,\ddot{\mathbf{u}}^n + \delta \ddot{\mathbf{u}}^{n+1}\right) \\ \mathbf{u}^{n+1} &= \mathbf{u}^n + \Delta t\,\dot{\mathbf{u}}^n + (\Delta t)^2\left(\left(\frac{1}{2} - \theta\right)\ddot{\mathbf{u}}^n + \theta \ddot{\mathbf{u}}^{n+1}\right) \end{aligned}$$

1st order if $\delta \neq \frac{1}{2}$
2nd order if $\delta = \frac{1}{2}$ and $\theta \neq \frac{1}{12}$
4th order if $\delta = \frac{1}{2}$ and $\theta = \frac{1}{12}$

Implementation of Newmark algorithm: the v-form

Find $\dot{\mathbf{u}}^{n+1}$ such that

$$\mathcal{A}_S\left(\dot{\mathbf{u}}^{n+1}, \mathbf{w}^S\right) = \mathcal{L}_S\left(\mathbf{w}^S\right) + \int_{\Gamma_0} \widehat{\boldsymbol{\eta}}^{n+1} \cdot \mathbf{w}^S \omega, \quad \forall \mathbf{w}^S = 0 \text{ on } \Gamma_D$$

$$\begin{aligned} \ddot{\mathbf{u}}^{n+1} &= \dots \\ \mathbf{u}^{n+1} &= \dots \end{aligned}$$

Time integration shema for fluid equations

Find $\mathbf{v}^{n+1}$ and p^{n+1} such that

$$\int_{\Omega^F_{t_{n+1}}} \rho^F \left(\frac{\mathbf{v}^{n+1} - \mathbf{V}^n}{\Delta t} \right) \cdot \mathbf{w}^F$$
$$+ \int_{\Omega^F_{t_{n+1}}} \rho^F \left(\left(\left(\mathbf{V}^n - \boldsymbol{\vartheta}^{n+1} \right) \cdot \nabla \right) \mathbf{v}^{n+1} \right) \cdot \mathbf{w}^F$$
$$+ a_F \left(\mathbf{v}^{n+1}, \mathbf{w}^F \right) + b_F \left(\mathbf{w}^F, p^{n+1} \right)$$
$$= \int_{\Omega^F_{t_{n+1}}} \mathbf{f}^F \cdot \mathbf{w}^F + \int_{\Sigma_1} \mathbf{h}^{n+1}_{in} \cdot \mathbf{w}^F + \int_{\Sigma_3} \mathbf{h}^{n+1}_{out} \cdot \mathbf{w}^F$$
$$- \int_{\Gamma_{t_{n+1}}} \boldsymbol{\eta}^{n+1} \cdot \mathbf{w}^F, \quad \forall \mathbf{w}^F \cdot \mathbf{n}^F = 0 \text{ on } \Sigma_2$$

$$b_F \left(\mathbf{v}^{n+1}, q \right) = 0, \quad \forall q$$

where $\mathbf{V}^n(\mathbf{x}) = \mathbf{v}^n \left(\mathcal{A}_{t_n} \circ \mathcal{A}^{-1}_{t_{n+1}}(\mathbf{x}) \right)$

Concise form

Find $\mathbf{v}^{n+1}$ and p^{n+1} such that

$$\begin{aligned}
\mathcal{A}_F\left(\mathbf{v}^{n+1}, \mathbf{w}^F\right) + b_F\left(\mathbf{w}^F, p^{n+1}\right) &= \mathcal{L}_F\left(\mathbf{w}^F\right) - \int_{\Gamma_{t_{n+1}}} \boldsymbol{\eta}^{n+1} \cdot \mathbf{w}^F, \\
&\qquad \forall \mathbf{w}^F \cdot \mathbf{n}^F = 0 \text{ on } \Sigma_2 \\
b_F\left(\mathbf{v}^{n+1}, q\right) &= 0, \quad \forall q
\end{aligned}$$

Semi-implicit time advancing scheme. From n to $n+1$

Step 1. Explicit prediction $\widetilde{\mathbf{u}}^{n+1} = \mathbf{u}^n + \Delta t\, \dot{\mathbf{u}}^n + \frac{(\Delta t)^2}{2} \ddot{\mathbf{u}}^n$
Step 2. Harmonic extension $\Delta \widetilde{\mathbf{d}}^{n+1} = 0$, $\widetilde{\mathbf{d}}^{n+1} = \widetilde{\mathbf{u}}^{n+1}$ on Γ_0,
$\widetilde{\mathbf{d}}^{n+1} = 0$ on $\Sigma_1 \cup \Sigma_2 \cup \Sigma_3$
Step 3. Build mesh $\widetilde{\mathcal{T}}_h^{n+1} = \mathbb{T}\left(\widehat{\mathcal{T}}_h\right)$, where $\mathbb{T}(\widehat{\mathbf{x}}) = \widehat{\mathbf{x}} + \widetilde{\mathbf{d}}^{n+1}(\widehat{\mathbf{x}})$
Step 4. Mesh velocity $\widetilde{\vartheta}^{n+1}(\mathbf{x}) = \frac{\widetilde{\mathbf{d}}^{n+1}(\widehat{\mathbf{x}}) - \widetilde{\mathbf{d}}^{n}(\widehat{\mathbf{x}})}{\Delta t}$

Step 5. $\mathbf{v}_{old}^S = \dot{\mathbf{u}}^n$, $\widehat{\boldsymbol{\lambda}} = \widehat{\boldsymbol{\eta}}^n$

Step 7. Solve fluid-structure problem by the Augmented Lagrangian Method in the fixed mesh $\widetilde{\mathcal{T}}_h^{n+1}$

Step 8. Update
$\mathbf{v}^{n+1} = \mathbf{v}^F$, $p^{n+1} = p^F$
$\widehat{\boldsymbol{\eta}}^{n+1} = \widehat{\boldsymbol{\lambda}}$
$\dot{\mathbf{u}}^{n+1} = \widehat{\mathbf{v}}^S$, $\ddot{\mathbf{u}}^{n+1} = \ldots$, $\mathbf{u}^{n+1} = \ldots$

Augmented Lagrangian Method

Step 7.1. Solve fluid problem: find $\mathbf{v}^F$ and p^F such that

$$\begin{aligned} \mathcal{A}_F\left(\mathbf{v}^F, \mathbf{w}^F\right) + b_F\left(\mathbf{w}^F, p^F\right) &= \mathcal{L}_F\left(\mathbf{w}^F\right) - \int_{\Gamma_{\widetilde{\mathbf{u}}}} \boldsymbol{\lambda} \cdot \mathbf{w}^F \\ &\quad - r \int_{\Gamma_{\widetilde{\mathbf{u}}}} \left(\mathbf{v}^F - \mathbf{v}^S_{old}\right) \cdot \mathbf{w}^F, \\ b_F\left(\mathbf{v}^F, q\right) &= 0, \end{aligned}$$

Step 7.2. Solve structure problem: find $\widehat{\mathbf{v}}^S$ such that

$$\mathcal{A}_S\left(\widehat{\mathbf{v}}^S, \mathbf{w}^S\right) = \mathcal{L}_S\left(\mathbf{w}^S\right) + \int_{\Gamma_0} \widehat{\boldsymbol{\lambda}} \cdot \mathbf{w}^S \omega + r \int_{\Gamma_0} \left(\widehat{\mathbf{v}}^F - \widehat{\mathbf{v}}^S\right) \cdot \mathbf{w}^S \omega$$

Step 7.3. If $\left\|\widehat{\mathbf{v}}^F - \widehat{\mathbf{v}}^S\right\|_{\Gamma_0} < tol$ **then break**
else $\widehat{\boldsymbol{\lambda}} = \widehat{\boldsymbol{\lambda}} + \rho\left(\widehat{\mathbf{v}}^F - \widehat{\mathbf{v}}^S\right)$; $\mathbf{v}^S_{old} = \widehat{\mathbf{v}}^S$; **goto Step 7.1.**

Physical and numerical parameters

The length is $L = 6\ cm$.
The inflow Σ_1 and outflow Σ_3 sections are segments of length 1.

The thickness of wall $h = 0.1\ cm$, the Young's modulus $E = 0.75 \cdot 10^6\ \frac{g}{cm \cdot s^2}$, the Poisson's ratio $\nu = 0.3$ and the mass density $\rho^S = 1.1\ \frac{g}{cm^3}$.

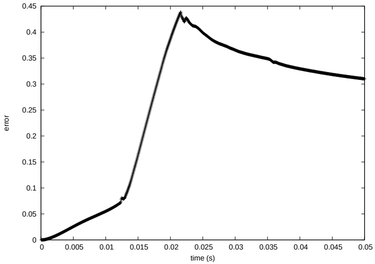
The viscosity of the fluid $\mu = 0.035\ \frac{g}{cm \cdot s}$, the volume force in fluid is $\mathbf{f}^F = (0,0)^T$, the outflow traction $\mathbf{h}_{out} = (0,0)^T$ and the inflow traction $\mathbf{h}_{in}(t) = \left(2000\left(1 - cos(\frac{2\pi t}{0.025})\right), 0\right)^T$ if $0 \leq t \leq 0.025$ and $\mathbf{h}_{in}(t) = (0,0)^T$ if $t > 0.025$.

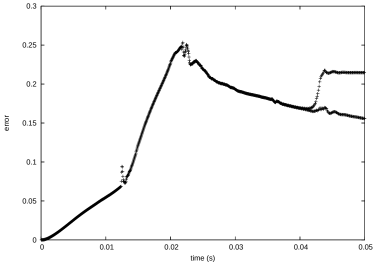
Newmark parameters: $\delta = 0.5$, $\theta = 0.25$

$\Delta t = 5 \cdot 10^{-5}$, $N = 1000$, $T = 0.05$
Penalization parameters $r = \rho = 10^4$

Triangular finite elements: *fluid* velocity $\mathbb{P}_1$+*bubble* and pressure $\mathbb{P}_1$, *structure*: $\mathbb{P}_1$

Computed $\left\|\widehat{\mathbf{v}}^F - \widehat{\mathbf{v}}^S\right\|_{\Gamma_0}$, when $r = \rho = 10^4$ (left) and $r = \rho = 2 \cdot 10^4$ (right)





No of iterations at one time step: 20, but the fluid and structure matrices do not change.

Concluding remarks

- Semi-implicit time advancing scheme is attractive.
- The equality of the velocities on the interface was treated by the Augmented Lagrangian Method.
- Investigate the dissipation source.
- Study the stability of the algorithm.