Rubrique

Semi-implicit algorithm for fluid-structure interaction in cerebral aneurysm

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RÉSUMÉ. Dans cet article, nous présentons un algorithme semi-implicite pour simuler le phénomène d'interaction fluide structure dans l'anévrisme cérébral. Le fluide est supposé visqueux, incompressible, gouverné par des équations de Navier-Stokes posées dans un domaine en mouvement. La structure est supposée gouvernée par le modèle de Saint-Venant Kirchhoff non linéaire, adopé pour des grands déplacements, mais avec petites déformations. A chaque pas de temps, un problème d'optimisation est résolu par une méthode des procédures partagées basée sur l'algorithme de BFGS (Broyden, Fletcher, Goldfard, Shano) pour satisfaire les conditions d'égalité des contraintes et de continuité des vitesses à l'interface. Les résutats numériques sont présentés.

ABSTRACT. This paper deals with a semi-implicit algorithm for solving fluid-structure interaction problem numerically holding in cerebral aneurysm. We assume that the fluid is governed by Navier Stokes equations setting in a moving domain and the structure is governed by nonlinear St-Venant Kirchholff elasticity model, which could be used for large displacements but small strain. At each time step, an optimization problem is solved by partitioned procedure method based in BFGS (Broyden, Fletcher, Goldfard, Shano) algorithm in order to get the continuity of stress as well as the continuity of velocity at the interface. The numerical results are presented.

MOTS-CLÉS : Fluide dans un domaine en mouvement, formulation ALE, élasticité non linéaire, procédures partagées, éléments finis, différences finies.

KEYWORDS : Fluid in a moving domain, ALE framework, nonlinear elasticity, partitioned procedures, finite element, finite difference

1. Introduction

The cerebral aneurysm is an abnormal dilation of a blood vessel wall under divers factors, like excess of tobacco and alcohol. It create therefore a pocket where the blood accumulate. The intercranial aneurysm is observed in the outer wall of curved vessel, it is found in the internal carotid artery near the apex of bifurcated vessels including the anterior communicating artery (see [1]). The discovery of cerebral aneurysm holds frequently between 35 years old to 60 years old (2% to 4% of the population and 3 women over 2 men). The unruptured aneurysm have been reported occur in up to 6% of the population and the aneurysm rupture causes over 90% of subarachnoid haemorrhages, which is associated to a high mortality rate (see [4]). There are some paper dealing with the interaction between the blood and the wall aneurysm (see [12], [11]). In [12], the authors, describe the flow dynamics and arterial wall interaction of a terminal aneurysm of simplified basilar artery and they compute its wall shear stress (WSS), pressure, effective stress and wall deformation. The geometry of cerebral aneurysm considered here is similar to one used in [10]. In this work, we present a fast semi-implicit algorithm for solving numerically the fluid-structure interaction arising in cerebral aneurysm. The term semi-implicit means that the velocity and the pressure of the fluid, the structure displacements are computed in implicit way, while the interface between the fluid and the structure is treated in explicit way. In [9], when the structure is governed by linear elasticity model, we have showed that this algorithm is unconditional stable. The algorithm implementation is done by partitioned procedures : at each time step, we firstly solve the structure problem by Newton's method to get the structure displacements and secondly, the fluid-structure problem is solved by a least square method based on the BFGS algorithm in order to get the continuity of the stress as well as the continuity of velocity at the interface. The major importance to work with this algorithm is that it use a fluid fixed mesh and an unique factorization of the fluid matrix during the optimization problem, that reduce considerably the time of computation. More precisly, in [7] from the CPU time computation we showed that the semi-implicit algorithm is **5.94** times faster than the implicit one.

2. Setting problem

Let us denote by Ω^S the undeformed structure domain bounded by : the rigid section Γ_0, the exterior section Γ_1 and the interior section Σ_0, and by Ω_0^F the initial fluid domain bounded by : the rigid section Σ_2, the inflow section Σ_1, the outflow section Σ_3 and the exterior boundary Σ_0, (see Figure 1).

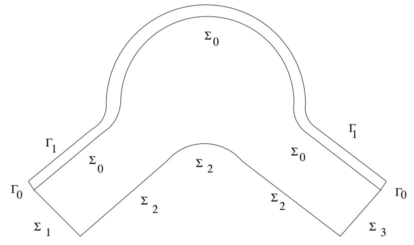

Figure 1. *The fluid-structure initial domain.*

The section Σ_0 represents the fluid-structure interface. Under the action of the fluid stress, the structure will be deformed. At the time instant t, the fluid occupies the domain Ω_t^F.

We assume that the fluid is governed by Navier-Stokes equations and the structure by the nonlinear St-Venant Kirchhoff elasticity model (see [3]). At each time $t \in [0,T]$, we are interested to know : the fluid velocity $\mathbf{v}(t) : \Omega_t^F \longrightarrow \mathbb{R}^2$, the fluid pressure $p(t) : \Omega_t^F \longrightarrow \mathbb{R}$ and the structure displacements $\mathbf{u}(t) : \Omega^S \longrightarrow \mathbb{R}^2$.

We introduce the ALE (Arbitrary Lagrangian Eulerian) coordinates in order to write the time derivative of fluid velocity with respect to a fix reference domain, see [8]. Let $\widehat{\Omega}^F$ be the reference fixed domain and let $\mathcal{A}_t, t \in [0,T]$ be a transformation such that :

$$\mathcal{A}_t(\widehat{\mathbf{x}}) = \widehat{\mathbf{x}},\ \forall\, \widehat{\mathbf{x}} \in \partial\Omega_t^F \setminus \Sigma_t,\ \mathcal{A}_t(\widehat{\Omega}^F) = \Omega_t^F,\ \mathcal{A}_t(\Sigma_0) = \Sigma_t,$$

where $\widehat{\mathbf{x}} = (\widehat{x}_1, \widehat{x}_2)^T \in \widehat{\Omega}^F$ are the ALE coordinates and $\mathbf{x} = (x_1, x_2)^T = \mathcal{A}_t(\widehat{\mathbf{x}})$ the Eulerian coordinates. We denote the fluid domain velocity by :

$$\boldsymbol{\vartheta}(\mathbf{x},t) = \frac{\partial \mathcal{A}_t}{\partial t}(\widehat{\mathbf{x}},t) = \frac{\partial \mathcal{A}_t}{\partial t}\Big(\mathcal{A}_t^{-1}(\mathbf{x})\Big)$$

and the ALE time derivative of the fluid velocity by : $\left.\dfrac{\partial \mathbf{v}}{\partial t}\right|_{\widehat{\mathbf{x}}}(\mathbf{x},t) = \dfrac{\partial \widehat{\mathbf{v}}}{\partial t}(\widehat{\mathbf{x}},t)$.

We assume that the fluid-structure interaction is governed by the following equations :

Navier-Stokes

$$\begin{aligned}
\rho^F\left(\left.\frac{\partial \mathbf{v}}{\partial t}\right|_{\widehat{\mathbf{x}}} + ((\mathbf{v}-\boldsymbol{\vartheta})\cdot\nabla)\,\mathbf{v}\right) - 2\mu^F\nabla\cdot\epsilon\,(\mathbf{v}) + \nabla p &= \mathbf{f}^F \ \text{ in } \Omega_t^F \times (0,T] && (1)\\
\nabla\cdot\mathbf{v} &= 0 \ \text{ in } \Omega_t^F \times [0,T] && (2)\\
\sigma^F\mathbf{n}^F &= \mathbf{h}_{in} \ \text{ on } \Sigma_1 \times (0,T] && (3)\\
\sigma^F\mathbf{n}^F &= \mathbf{h}_{out} \ \text{ on } \Sigma_3 \times (0,T] && (4)\\
\mathbf{v} &= 0 \ \text{ on } \Sigma_2 \times (0,T] && (5)\\
\mathbf{v}(X,0) &= \mathbf{v}^0(X) \ \text{ in } \Omega_0^F && (6)
\end{aligned}$$

where

$$\sigma^F = -p\mathbb{I}_2 + 2\mu^F\epsilon(\mathbf{v}),\ \text{ with } \epsilon(\mathbf{v}) = \frac{1}{2}\Big(\nabla\mathbf{v} + (\nabla\mathbf{v})^T\Big),$$

$\mathbf{f}^F = (f_1^F, f_2^F)$ are the applied forces, $\mathbf{h}_{in}$ and $\mathbf{h}_{out}$ are the prescribed boundary stresses on Σ_1 and on Σ_3, ρ^F and μ^F are the mass density and the viscosity of the fluid.

Nonlinear elasticity equations

$$\begin{aligned}
\rho^S\frac{\partial^2 u}{\partial t^2} - \nabla\cdot\sigma^S &= \mathbf{f}^S,\ \text{ in } \Omega^S \times (0,T] && (7)\\
\mathbf{u} &= 0,\ \text{ on } \Gamma_0 \times (0,T] && (8)\\
\sigma^S\mathbf{n}^S &= 0,\ \text{ on } \Gamma_1 \times (0,T] && (9)\\
\mathbf{u}(X,0) &= \mathbf{u}^0(X),\ \text{ in } \Omega^S && (10)\\
\frac{\partial \mathbf{u}}{\partial t}(X,0) &= \dot{\mathbf{u}}^0(X),\ \text{ in } \Omega^S, && (11)
\end{aligned}$$

where $\rho^S > 0$ is the mass density of the structure, μ^S, and λ^S are the Lamé's coefficients, $\mathbf{f}^S = (f_1^S, f_2^S)$ are the applied forces and where the stress tensor σ^S is given by :

$$\sigma^S = (\mathbb{I}_2+\nabla\mathbf{u})\big(\lambda^S(tr(E(\mathbf{u}))+2\mu^S\mathbb{E}(\mathbf{u})\big),\ \text{ with } \mathbb{E}(\mathbf{u}) = \frac{1}{2}\Big(\nabla\mathbf{u}+(\nabla\mathbf{u})^T+(\nabla\mathbf{u})^T\nabla\mathbf{u}\Big).$$

Interface conditions

$$\mathbf{v}(X+\mathbf{u}(X,t),t) = \frac{\partial \mathbf{u}}{\partial t}(X,t), \quad \text{on } \Sigma_0 \times (0,T] \tag{12}$$

$$(\sigma^F \mathbf{n}^F)_{(X+\mathbf{u}(X,t),t)}\omega = -(\sigma^S \mathbf{n}^S)_{(X,t)}, \quad \text{on } \Sigma_0 \times (0,T], \tag{13}$$

where $\omega = \|\text{cof}(\nabla \mathbb{T}_u)\mathbf{n}^S\|_{\mathbb{R}^2}$, and $\mathbb{T}_u : \Gamma_0 \longrightarrow \Gamma_t$ defined by : $\mathbb{T}_u(X) = X + \mathbf{u}(X,t)$ and $\mathbf{n}^S = (n_1^S, n_2^S)$ is the unit outward normal to Σ_0.

Remark : The existence and uniqness of the weak solutions of (1)-(13) can be showed using the same theory as in [2]. For the coupling problem, some regularity are requiert, for example the condition (12) make sens if the structure velocity is in $(H^1(\Omega^S))^2$.

3. Discretization and weak formulation for the fluid equations

Let $N \in \mathbb{N}^*$ be the number of time steps and $\Delta t = T/N$ the step time. We set $t_n = n\Delta t$ for $n = 0, \cdots, N$ the subdivision of $[0,T]$. We denote by $\mathbf{f}^n$, $\mathbf{h}_{in}^n$, $\mathbf{h}_{out}^n$, p^n, $\mathbf{v}^n$ the time approximation of $\mathbf{f}^F$, $\mathbf{h}_{in}$, $\mathbf{h}_{out}$, p, $\mathbf{v}$ respectively. We consider an implicit Euler scheme for the time derivative and a linearization of the convective term. We set $\widehat{\Omega}^F = \Omega_n^F$ and we define $\boldsymbol{\vartheta}^n = (\vartheta_1^n, \vartheta_2^n)$ the velocity of the fluid domain as solution of :

$$\Delta_{\widehat{\mathbf{x}}} \boldsymbol{\vartheta}^n = 0 \text{ in } \Omega_n^F, \quad \boldsymbol{\vartheta}^n = 0 \text{ on } \partial\Omega_n^F \setminus \Sigma_n, \quad \boldsymbol{\vartheta}^n = \mathbf{v}^n, \text{ on } \Sigma_n.$$

For all $n = 0, \cdots, N-1$, we define the discrete ALE map $\mathcal{A}_{t_{n+1}} : \Omega_n^F \longrightarrow \mathbb{R}^2$ by :

$$\mathcal{A}_{t_{n+1}}(\widehat{x}_1, \widehat{x}_2) = (\widehat{x}_1 + \Delta t \vartheta_1^n, \widehat{x}_2 + \Delta t \vartheta_2^n).$$

We set $\Omega_{n+1}^F = \mathcal{A}_{t_{n+1}}(\Omega_n^F)$ and $\Sigma_{n+1} = \mathcal{A}_{t_{n+1}}(\Sigma_n)$.
We define the map $\mathbb{T} = \mathcal{A}_{t_n} \circ \mathcal{A}_{t_{n-1}} \circ \cdots \circ \mathcal{A}_{t_1}$ and we can observe that $\Sigma_n = \mathbb{T}(\Sigma_0)$.

We define the fluid velocity $\mathbf{v}^{n+1} : \Omega_{n+1}^F \longrightarrow \mathbb{R}^2$ (respectively the fluid pressure $p^{n+1} : \Omega_{n+1}^F \longrightarrow \mathbb{R}$) at time instant $(n+1)$ on Ω_{n+1}^F by :

$$\mathbf{v}^{n+1}(\mathbf{x}) = \widehat{\mathbf{v}}^{n+1}(\widehat{\mathbf{x}}), \quad p^{n+1}(\mathbf{x}) = \widehat{p}^{n+1}(\widehat{\mathbf{x}}) \; \forall \, \widehat{\mathbf{x}} \in \widehat{\Omega}_n^F \text{ and } \mathbf{x} = \mathcal{A}_{t_{n+1}}(\widehat{\mathbf{x}}). \tag{14}$$

We introduce the following spaces of test function :

$$\widehat{W}_n^F = \{\widehat{\mathbf{w}}^F \in (H^1(\Omega_n^F))^2; \; \widehat{\mathbf{w}}^F = 0 \text{ on } \Sigma_2\}, \quad \widehat{Q}_n^F = L^2(\Omega_n^F).$$

From the Green's formula we get the following discrete weak form of (1)-(6) :
Find $\widehat{\mathbf{v}}^{n+1} \in \widehat{W}_n^F, \widehat{p}^{n+1} \in \widehat{Q}_n^F$, with $\widehat{\mathbf{v}}^{n+1} \circ \mathbb{T} = \dfrac{\mathbf{u}^{n+1} - \mathbf{u}^{n-1}}{2\Delta t}$, on Σ_0 such that :

$$\rho^F \int_{\Omega_n^F} \frac{(\widehat{\mathbf{v}}^{n+1} - \mathbf{v}^n)}{\Delta t} \cdot \widehat{\mathbf{w}}^F + \rho^F \int_{\Omega_n^F} (((\mathbf{v}^n - \boldsymbol{\vartheta}^n) \cdot \nabla_{\widehat{\mathbf{x}}})\widehat{\mathbf{v}}^{n+1}) \cdot \widehat{\mathbf{w}}^F$$
$$+2\mu^F \int_{\Omega_n^F} \epsilon_{\widehat{\mathbf{x}}}(\widehat{\mathbf{v}}^{n+1}) : \epsilon_{\widehat{\mathbf{x}}}(\widehat{\mathbf{w}}^F) - \int_{\Omega_n^F} \widehat{p}^{n+1}(\nabla_{\widehat{\mathbf{x}}} \cdot \widehat{\mathbf{w}}^F) - \int_{\Omega_n^F} \widehat{q}(\nabla_{\widehat{\mathbf{x}}} \cdot \widehat{\mathbf{v}}^{n+1})$$
$$= \int_{\Omega_n^F} \widehat{\mathbf{f}}^{n+1} \cdot \widehat{\mathbf{w}}^F + \int_{\Sigma_n} (\sigma^F \mathbf{n}^F) \cdot \widehat{\mathbf{w}}^F + \int_{\Sigma_2} \mathbf{h}_{in}^{n+1} \cdot \mathbf{w}^F + \int_{\Sigma_3} \mathbf{h}_{out}^{n+1} \cdot \mathbf{w}^F, \tag{15}$$

for all $\widehat{\mathbf{w}}^F \in \widehat{W}_n, \; \widehat{q} \in \widehat{Q}_n^F$ with $\widehat{\mathbf{f}}^{n+1} = \mathbf{f}^{n+1} \circ \mathcal{A}_{t_{n+1}}$.

4. Discretization and weak formulation of structure equations

For the structure equations, we denote by $\mathbf{u}^n$, $\mathbf{g}^n$ the time approximation of $\mathbf{u}$, $\mathbf{f}^S$. We set $\mathbf{F}^n = (\sigma^S \mathbf{n}^S)(t_n)$. We use a θ-centred scheme of second order in time, with $1/4 \leq \theta \leq 1/2$. We define the space of test functions :

$$\mathbf{W}^S = \{\mathbf{w}^S \in (H^1(\Omega^S))^2, \quad \mathbf{w}^S = 0 \text{ on } \Gamma_0\}.$$

From the Green formula, we get the following discrete weak form of (7)-(11) :
Find $\mathbf{u}^{n+1} \in \mathbf{W}^S$ such that :

$$\begin{aligned}
&\int_{\Omega^S} \rho^S \frac{\left(\mathbf{u}^{n+1} - 2\mathbf{u}^n + \mathbf{u}^{n-1}\right)}{(\Delta t)^2} \cdot \mathbf{w}^S d\mathbf{X} + \theta a_S\left(\mathbf{u}^{n+1}, \mathbf{w}^S\right) + (1-2\theta) a_S\left(\mathbf{u}^n, \mathbf{w}^S\right) \\
&+\theta a_S\left(\mathbf{u}^{n-1}, \mathbf{w}^S\right) = \int_{\Omega^S} \left(\theta \mathbf{f}^{S,n+1} + (1-2\theta)\mathbf{f}^{S,n} + \theta \mathbf{f}^{S,n-1}\right) \cdot \mathbf{w}^S d\mathbf{X} \\
&+ \int_{\Gamma_0} \left(\theta \mathbf{F}^{n+1} + (1-2\theta)\mathbf{F}^n + \theta \mathbf{F}^{n-1}\right) \cdot \mathbf{w}^S d\, s, \quad \forall \mathbf{w}^S \in \mathbf{W}^S, \qquad (16)
\end{aligned}$$

where $a_S\left(\mathbf{u}, \mathbf{w}^S\right) = \int_{\Omega^S} \sigma^S(\mathbf{u}) : \left(\nabla \mathbf{w}^S\right) d\mathbf{X}$.

4.1. Newton's method for the structure equations

The tensor $\sigma^S(\mathbf{u})$ is nonlinear, we use the Newton method to compute the structure displacements. From the derivative with respect to $\mathbf{u}$ of the components of $\mathbb{E}(\mathbf{u})$ for an arbitrary $\mathbf{h} = (h_1, h_2) \in W^S$, we can compute : $\dfrac{da_S}{d\mathbf{u}}(\mathbf{u}, \mathbf{w}^S)\mathbf{h} = \displaystyle\sum_{i,j=1}^{2} \int_{\Omega^S} \frac{d\sigma_{ij}^S}{d\mathbf{u}}(\mathbf{u})\mathbf{h} \frac{\partial \omega_i^S}{\partial x_j}$

Newton Algorithm

Step 0. Initialization. Set $k = 0$ and $\mathbf{u}^{n+1,0} = \mathbf{u}^n$. We generate $\mathbf{u}^{n+1,k}$ for $k = 1, 2, \cdots$
Step 1. Find $\mathbf{h}^k$ the solution of the linear system

$$\begin{aligned}
&\int_{\Omega^S} \rho^S \frac{\mathbf{h}^k}{(\Delta t)^2} \cdot \mathbf{w}^S dX + \frac{da^S}{d\mathbf{u}}(\mathbf{u}^{n+1,k}, \mathbf{w}^S)\mathbf{h}^k \\
&= \int_{\Omega^S} \rho^S \frac{(\mathbf{u}^{n+1,k} - 2\mathbf{u}^n + \mathbf{u}^{n-1})}{(\Delta t)^2} \cdot \mathbf{w}^S dX \\
&+\theta a_S(\mathbf{u}^{n+1,k}, \mathbf{w}^S) + (1-2\theta) a_S(\mathbf{u}^n, \mathbf{w}^S) + \theta a_S(\mathbf{u}^{n-1,k}, \mathbf{w}^S) \qquad (17) \\
&- \int_{\Omega^S} (\theta \mathbf{f}^{S,n+1} + (1-2\theta)\mathbf{f}^{S,n} + \theta \mathbf{f}^{S,n-1}) \cdot \mathbf{w}^S dX \\
&- \int_{\Omega^S} (\theta \mathbf{F}^{n+1} + (1-2\theta)\mathbf{F}^n + \theta \mathbf{F}^{n-1}) \cdot \mathbf{w}^S dX, \quad \forall\, \mathbf{w}^S \in W^S,
\end{aligned}$$

Step 2. If $\mathbf{h}^k$ is small, then stop.
Step 3. Set $\mathbf{u}^{n+1,k+1} = \mathbf{u}^{n+1,k} - \mathbf{h}^k$; $k \longleftarrow k+1$; go to **Step 1**.

The finite element method is used to solve the variational equation (17). We denote by $\mathbf{u}_h^{n+1}$ the solution which correspond to the structure displacement at time t_{n+1}.

5. Semi-implicit algorithm for the coupling problem

We mean by semi-implicit the fact that the interface position is computed explicitly, while the displacements, velocity and pressure are computed implicitly. An optimization problem must be solved in order to get the continuity of stress at the interface. The stress $\mathbf{F}^{n+1} = (\sigma^S \mathbf{n}^S)(t_{n+1})$ is unknown. We approach it by : $\mathbf{F}^{n+1} = \sum_{i=1}^{m} \xi_i^{n+1} \boldsymbol{\psi}^i$, where ξ_i^{n+1} have to be identified, $\boldsymbol{\psi}^i \in \left(L^2\left(\Gamma_0\right)\right)^2$ are shape functions. (see [7]).

Semi-implicit algorithm

Step 1. Compute the mesh velocity $\boldsymbol{\vartheta}^n$.

Step 2. Assembling the finite element matrix of fluid problem using the frozen mesh $\mathcal{T}^n$. Get a LU factorization of the matrix.

Step 3. Solve the fluid-structure problem using the fluid mesh $\mathcal{T}^n$ by BFGS algorithm :

$$\xi^{n+1} \in \arg\min_{\xi \in \mathbb{R}^m} J(\xi),$$

where the cost function J is computed as following :

1) Solve (17) by Newton Method under the load $\mathbf{F}^{n+1} = \sum_{i=1}^m \xi_i^{n+1} \boldsymbol{\psi}^i$ and get the displacement $\mathbf{u}^{n+1}$.

2) Solve (15) on the mesh $\mathcal{T}^n$ under prescribed velocity at the interface $\widehat{\mathbf{v}}^{n+1} = \dfrac{\mathbf{u}^{n+1} - \mathbf{u}^{n-1}}{2\Delta t}$, in order to get the fluid velocity $\widehat{\mathbf{v}}^{n+1}$ and $\widehat{p}^{n+1}$.

3) Compute $\alpha_i = \displaystyle\int_{\Gamma_0} \left(\sum_{j=1}^m \xi_j \boldsymbol{\psi}^j \right) \cdot \boldsymbol{\psi}^i d\, s, \ \beta_i = - \int_{\Gamma_0} \left(\sigma^F \mathbf{n}^F\right)_{(\mathbf{X}+\mathbf{u}(\mathbf{X},t),t)} \cdot \boldsymbol{\psi}^i d\, s.$

4) Set the cost function $J(\boldsymbol{\alpha}) = \dfrac{1}{2} \|\boldsymbol{\alpha} - \boldsymbol{\beta}\|^2_{\mathbb{R}^m}$.

Step 4. Build mesh $\mathcal{T}^{n+1} = \mathcal{A}_{t_{n+1}}(\mathcal{T}^n)$ and save $\mathcal{T}^{n+1}$, $\mathbf{v}^{n+1}$, p^{n+1} given by (14).

6. Numerical results

6.0.1. Physical parameters

The length of Σ_1 and of Σ_3 is 3 mm, the length of Σ_2 is 5 mm. The interface Σ_0 is composed by two segments of length 5 mm and an arc of diameter 6 mm. The fluid viscosity is $\mu = 0.003 \ \frac{g}{cm \cdot s}$, the fluid density is $\rho^F = 1 \ \frac{g}{cm^3}$ and the volume forces are $\mathbf{f}^F = (0,\, 0)^T$. The prescribe boundary stress on Σ_3 is $\mathbf{h}_{out}(x,t) = (0,\, 0)^T$ and on Σ_1 is $\mathbf{h}_{in}(x,t) = (10^3(1 - \cos(2\pi t/0.025)),\, 0)^T$, if $x \in \Sigma_2,\ 0 \le t \le 0.025$ and $\mathbf{h}_{in}(x,t) = (0,\, 0)^T$, if $x \in \Sigma_2,\ 0.025 \le t \le T$.
The up boundary Γ_3 is an arc with diameter 6 mm, the length of Γ_1 and Γ_2 is 0.3 mm. The Young modulus is $E = 3 \cdot 10^6 \ \frac{g}{cm \cdot s^2}$, the Poisson ratio is $\nu = 0.3$, the structure mass density is $\rho^S = 1.1 \frac{g}{cm^3}$ and the volume force are $\mathbf{f}^S = (0,\, 0)^T$. The Lamé's coefficients are computed by the formulas : $\lambda^S = \dfrac{\nu^S E}{(1 - 2\nu^S)(1 + \nu^S)}, \quad \mu^S = \dfrac{E}{2(1 + \nu^S)}$.

6.0.2. Numerical parameters

The numerical tests have been performed using FreeFem++ (see [5]). We have used for the structure a reference mesh of 60 triangles and 62 vertices and for the fluid a reference mesh of 1615 triangles and 881 vertices. For the approximation in space of the fluid velocity and pressure, we have used the triangular finite element $\mathbb{P}_1 + bubble$ and $\mathbb{P}_1$ respectively. The finite element $\mathbb{P}_1$ was employed for the displacements of the structure.

6.0.3. Behavior of solutions

We show in the figure (2) the behavior of normal component of WSS at three different points on the interface. Near the inflow section, the maximal value of WSS is around $150\, dyn/cm^2$, near the apex, this value is around $50\, dyn/cm^2$ and near the outflow section, the value is about $100\, dyn/cm^2$. These results are comfirmed by [11].
The figure (3) shows the fluid and structure meshes (at top left), the behvior of the fluid velocity (at bellow left), of the fluid pressure (at top right) and of the structure velocity (at bellow right). We can observe that, the compatibility of meshes are not necessary verified at the interface.

7. Conclusion

In this paper, a semi-implicit algorithm based on the strategies developed in [9] and [7] has been used to simulate the fluid-structure interaction in cerebral aneurysm. The Newton method is used to solve the nonlinear model of the structure. At each time step, an optimization problem is solved by partitioned procedure based on BFGS algorithm in order to get the continuity of velocity as well as the continuity of the stress at the interface.

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9. Figures

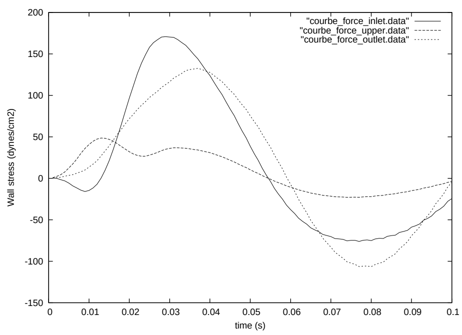

Figure 2. *Normal component of the wall shear stress at three points on the interface.*

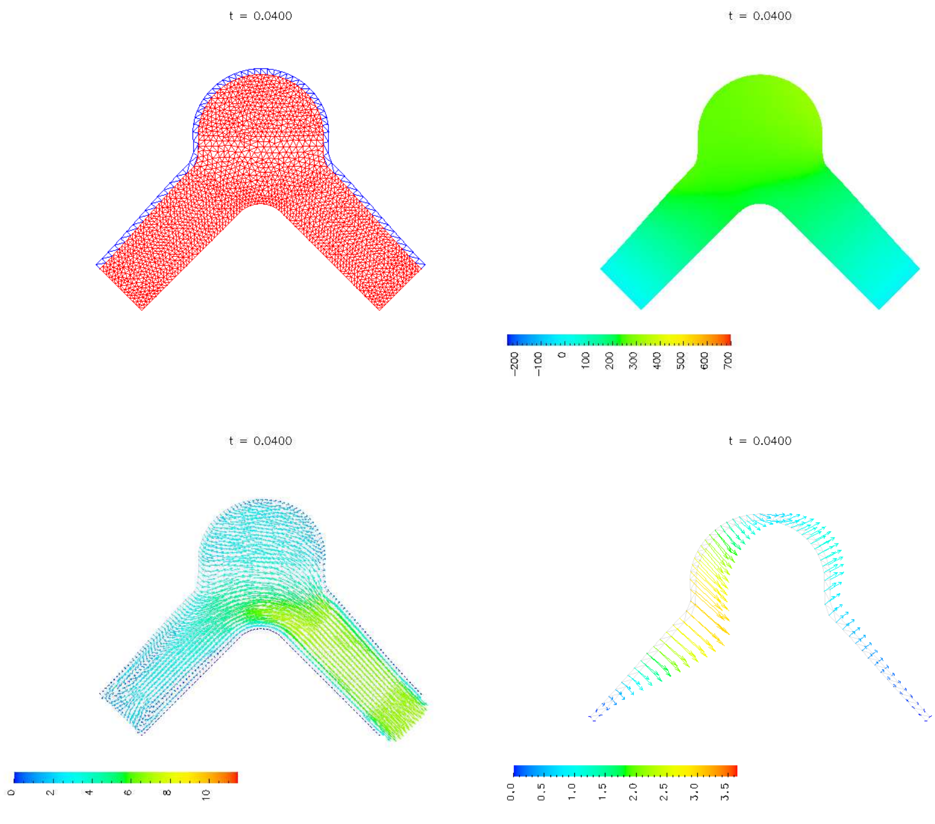

Figure 3. *Fluid-structure meshes, fluid pressure, fluid velocities and structure velocities at time instant* $t = 0.040\,s$.